APRIL/MAY 2024

# DMA43/GMA43 — FUNCTIONAL ANALYSIS

Time : Three hours Maximum : 75 marks

Shanmuga Industries Arts & Science College T.V.Malai

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Prove or disprove: Any vector space $N \neq \{0\}$ with discrete metric is not a normed linear space.

2. Discuss the convexity of C [0, 1].

3. Is $l_1$, the conjugate space of $l_x$? Justify.

4. Make use of dense sets, classify that the boundedness of an operator does not imply that is closed.

5. Apply Schwarz's inequality and derive Bessel's inequality.

6. Distinguish orthonormal and complete orthonormal sets.

7. How would you analyze the space $l_p$ with $p \neq 2$ is not an inner product space?



SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. State and prove Hahn Banach Theorem.

17. State and Prove Open Mapping Theorem.

18. Explain Schwarz inequality in detail and identify when equality holds.

19. Distinguish Normal and Unitary operators with proper arguments.

20. How can you prove that spectrum of an element in a Banach Algebra is non-empty.

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8. Let T on $l_2$ as $T_x = (0, x_1, x_2, ....)$ for every $x = (x_n) \in l_2$ Find the adjoint of T.

9. Find the spectrum of T, where $T : l_2 \to l_2$ defined by $T_x = \left[x_1, \frac{x_2}{2}, ...., \frac{x_i}{i} ....\right]$.

10. Validate the statement: Every divisor of zero is a topological divisor of zero.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) Show that on a finite dimensional space, all norms are equivalent.

Or

(b) Let $N$ and $N'$ be normed linear spaces and let $T : N \to N'$ be a bounded linear transformation. If $M$ is the kernel of $T$, then show that $T$ induces a natural transformation $T'$ of N/M onto $N'$ such that $\|T'\| = \|T\|$.

12. (a) Examine that the closedness of the graph of a linear transformation between Banach spaces is necessary and sufficient for the boundedness of the transformation.

Or



(b) How would you classify collection wise uniform boundedness of bounded operators deduced from the pointwise boundedness of such operators.

13. (a) Is the norm on a Banach space satisfy the parallelogram law? Justify your answer.

Or

(b) Validate the statement : If we drop the assumption closed subspace or complete whole space in the orthogonal decomposition theorem.

14. (a) Explain how a self-adjoint operator becomes a normal operator.

Or

(b) Explain the condition under which the sum of finite number of projections is a projection.

15. (a) Compute the relation connecting set of all regular and singular elements.

Or

(b) Derive spectral radius formula make use of Laurent's expansion.